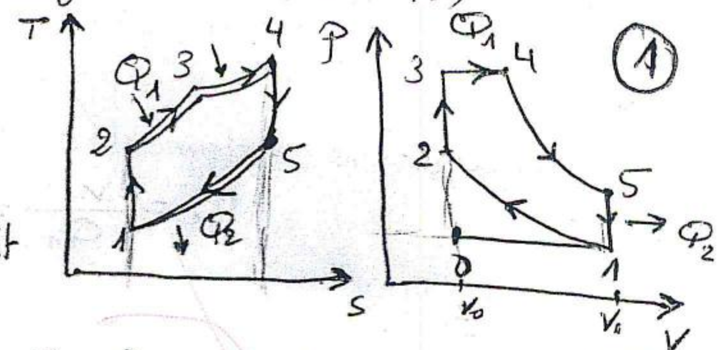
Corrige type (Examen MIH)

Exo 1 :

(1) Représenter le cycle dans les diagramme (P,V) et (T,S)



(1)

(2) Calculer la masse de gaz au cours du cycle

Le gaz est supposé comme un gaz parfait

$\Rightarrow P_1V_1 = nRT_1 \Rightarrow n = \frac{P_1V_1}{RT_1}$

et $m = nM = \frac{P_1V_1}{RT_1}.M$: AN $m = \frac{10^5 . 3.10^{-3}}{8,314.(50+273)}.30,7 = 3,429 \simeq \boxed{3,43g}$ (1

(3) Calculer les variables P, V et T aux points 2, 3, 4 et 5

au point (1) on a : $P_1 = 1$ bar ; $T_1 = 50C° = 323K°$ et $V_1 = 3\ell = 3.10^{-3} m^3$

au point (2) on a $a = \frac{V_1}{V_0} = \frac{V_1}{V_2} = 6 \Rightarrow V_2 = \frac{V_1}{6} = \frac{3}{6} = \boxed{0,5\ell}$

1 → 2 transformation isentropique

$$P_1V_1^\gamma = P_2V_2^\gamma \Rightarrow P_2 = \left(\frac{V_1}{V_2}\right)^\gamma P_1 = a^\gamma P_1 = 6^{1,35} . 1 = \boxed{11,23 \text{ bar}}$$

$$P_2V_2 = \frac{m}{M}RT_2 \Rightarrow T_2 = \frac{P_2V_2}{R}\frac{M}{m} = \frac{(0,5.10^{-3})(11,23.10^5)}{8,314}\frac{30,7}{3,43} = \boxed{604,48K°}$$

au point (3) on a : 2 → 3 transformation isochore $\Rightarrow V_3 = V_2 = \boxed{0,5\ell}$

3 → 4 〃 isobare $\frac{V_3}{V_4} = \frac{T_3}{T_4} \Rightarrow T_4 = bT_3$

D'après le bilan $Q_1 = m\,C_v(T_3 - T_2) + m\,C_p(bT_3 - T_3) \Rightarrow T_3 = \frac{Q_1 + m\,C_v T_2}{m[(b-1)C_p + C_v]}$

$$C_v = \frac{R}{(\gamma-1)M} = \frac{8,314}{(1,35-1).30,7.10^{-3}} = 773,75 \text{ J/kg K°}$$

$$C_p = \gamma C_v = 1,35 . 773,75 = 1044,57 \text{ J/kgK°}$$

$$T_3 = \frac{6277 + 3,43.10^{-3}.773,75.604,48}{3,43.10^{-3}[(1,4-1).1044,57 + 773,75]} = \boxed{1928,32K°}$$

$$P_3 = \frac{P_2T_3}{T_2} = \frac{11,23.1928,32}{604,48} = \boxed{35,82 \text{ bar}}$$

au point (4) on a 3 → 4 transformation isobare $P_4 = P_3 = \boxed{35,82 \text{ bar}}$

$$T_4 = bT_3 = 1,4.1928,32 = \boxed{2699,64 K°}$$

$$V_4 = bV_2 = 1,4.0,5 = \boxed{0,7\ell}$$

au point (5) on a 4 → 5 transformation adiabatique et $V_5 = V_1 = \boxed{3\ell}$

$$P_4V_4^\gamma = P_5V_5^\gamma \Rightarrow P_5 = P_4\left(\frac{V_4}{V_1}\right)^\gamma = 35,82\left(\frac{0,7}{3}\right)^{1,35} = \boxed{5,022 \text{ bar}}$$

$$T_4V_4^{\gamma-1} = T_5V_5^{\gamma-1} \Rightarrow T_5 = T_4\left(\frac{V_4}{V_5}\right)^{\gamma-1} = 2699,64\left(\frac{0,7}{3}\right)^{0,35} = \boxed{1622,17K°}$$

Tableau récapitulatif

		2	3	4	5	
P	bar	11,23	35,82	35,82	5,022	(1)
V	ℓ	0,5	0,5	0,7	3	(1)
T	K°	604,48	1928,32	2699,64	1622,17	(1)

(4) Calculer le travail fourni par le cycle :

$$\Delta U_{cycle} = W + Q_T = 0 \Rightarrow W = -Q_T$$

$$Q_T = \cancel{Q_{12}}^{0} + (Q_{23}+Q_{34}) + \cancel{Q_{45}}^{0} + Q_{51} = Q_1 + Q_{51} = Q_1 + m c_v (T_1 - T_5)$$

$$Q_{51} = m c_v (T_1 - T_5) = 3{,}43 \cdot 10^{-3} \cdot 773{,}75 \cdot (323 - 1622{,}17) = -3447{,}94\ J$$

$$W = Q_1 + Q_{51} = 6277 - 3447{,}94 = \boxed{2829{,}05\ J}\ (1)$$

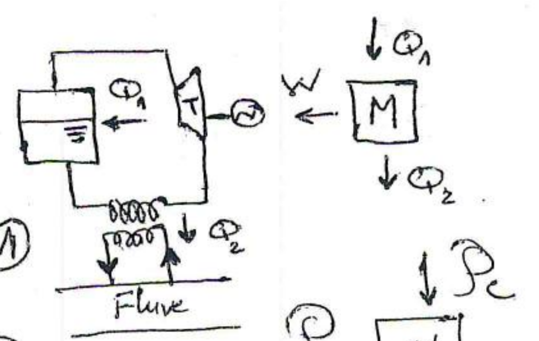
(5) Calculer le rendement :

$$\eta_c = \frac{W}{Q_1} = \frac{2829{,}05}{6277} = 0{,}4507 = \boxed{45{,}07\%}\ (1)$$

Exo 2 :

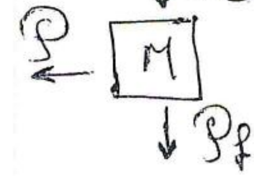
(1) le rendement de Carnot (1)

$$\eta_c = \frac{T_c - T_f}{T_c} = 1 - \frac{T_f}{T_c} = 1 - \frac{293}{593} = \boxed{50{,}6\%}$$



(2) le rendement réel et les puissances

$$\eta_{réel} = 0{,}6\eta_c = 0{,}6 \cdot 0{,}506 = \boxed{30{,}3\%}\ (1)$$



Puissance transférée a la source chaude $P_c = \frac{dQ_c}{dt}$

" " " froide $P_f = \frac{dQ_f}{dt}$

$$Q_c = \dot m C \Delta T \Rightarrow |P_c| = \frac{dQ_c}{dt} = \frac{dW}{dt}\cdot\frac{1}{\eta_{réel}} = \frac{P}{\eta_{réel}} = \frac{1}{0{,}303} = 3{,}3\ MW$$

ou : $Q_c = \frac{|-W|}{\eta_{réel}} \Rightarrow P_c = \frac{dW}{dt}\cdot\frac{1}{\eta_{réel}} = \frac{P}{\eta_{réel}} = \boxed{3{,}3\ MW}$ (1,5)

Calcul P_f : on a $\Delta U_{cycle} = W + Q_c + Q_f = 0$

$$\Rightarrow \frac{dW}{dt} + \frac{dQ_c}{dt} + \frac{dQ_f}{dt} = 0$$

$$P + P_c + P_f = 0 \Rightarrow P_f = -P - P_c$$

$$P_f = -P_c - P = 3{,}3 - 1 = \boxed{2{,}3\ MW}\ (1,5)$$

(3) La variation de Temperature

$$Q_f = m c \Delta T \text{ et } P_f = \frac{dQ_f}{dt} = \frac{d}{dt}(m c \Delta T) = \dot m c \Delta T$$

$$\Rightarrow \Delta T = \frac{P_f}{\dot m C} = \frac{2{,}3 \cdot 10^6}{(150 \cdot 10^3)\, 4185} = \boxed{3{,}7 \cdot 10^{-3}\ C°}\ (1)$$